Algebraic Fundamental Laws About Time Variable in Electric Networks

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Contents

[0] Preliminary note : This paper is intended to construct a quite new system of theory in order to manage electromagnetic phenomena more freely about time parameter.

In order to foresee the contour of the theory as clearly as possible and to give generality to it, we investigated it from the various sides of mathematical physics and mathematics. The philosophy of our research in [1] seemed necessary for this purpose.

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Whether our manner of construction of the theory is adequate or not is judged by that whether it promotes the freedom of managing the phenomena or not and on the other whether it has a wide generality for the other phenomena or not.

Chapter I The philosophy of our research.

a) first direct motive : requirement of the theory of the pulse phenomena.

[1.1] Hitherto in electric engineering, both in communication and power engineering, alternating current had been mainly practised such as audio-frequency technique, carrier communication, radio in communication and on the other power generation, transmission and distribution in power engineering.

Therefore also the theory has been developed and popularized mainly in the domain of alternating current.

[1.2] But recently pulse phenomena or phenomena in which pulse plays main role are gradually increasing, for example from the age of Morse telegraph and teletype to telephotography, television, time division multiplex communication and pulse tester and radar in the communication engineering, and transients in the direct current power transmission and pulse tester in the power line.

[1.3] Also in the other fields of physics, e. g. supersonic pulse testers for measuring the depth of the water or thickness of metallic plates or pipes are popularized.

Theory for these seems not so convenient as in the Newtonian mechanics for investigation of motions of the bodies in the classical physics.

[1.4] For these pulse phenomena, we have fetched their theory far to the a. c. theory by the Fourier Integral because of the inertia of the advancement of the a. c. theory. So there does not exist plain algebraic theory comparable with the a. c. theory except the complicated "Heaviside's operational calculus" which necessitates vast culture of the Laplace transform, namely integral in the domain of the complex variable and therefore the theory of function of the complex variable to obtain accurate results. (for example GBT, see the end)

[1.5] Even in this symbolic method, in individual problem the main process in each calculation is arithmetic and algebraic and it does not contain proper operations of the complex function theory nor operations of the serious "Limes" (limitting process).

[1.6] "Simple laws for simple phenomena" has been our motto, because for the simple phenomena simple quantitative laws should be obtained by the direct induction from the experiments. The above symbolic method seems against this principle.

[1.7] Above tendency of practice and lack of plain theory for them urged us to investigate a mathematical possibility of construcring a purely algebraic laws of transient phenomena in the electric networks.

[1.8] The above necessity can be symbolized to construct "x" in

wave : pulse = a. c. theory : x
(phenomena) (algebra) (1.1)

or

wave mechanics : quantum mechanics = a. c. theory : x
(physics) (electric engineering) (1.2)

b) mathematical investigation

[1.9] When we require quantitative result of high accuracy, then we represent it as tables of numericals and not as curves nor graphs. From this fact we can conclude that the concept "continuous" is a mere geometrical intuition and the whole important ultimate result obtained from the world of logic in the mathematics, namely from the all finite analysis is axiomatically obtained by treating the object as finite and discrete and by finite process.

[1.10] It is invariant till now that the fundamental principle to apply every logical analysis to the continuum is the "axiom of Archimedes" or axiom of measurement (the fifth axiom of continuum in the Euclidean geometry in HGG § 8. PSH. chap. 3, PDP. app. chap. 1), [1.20], [1.27] are multi-dimensional application.

[1.11] The pure mathematicians investigate the construction and substance of the continuum or endeavour to solve the problem of density of the continuum. (e. g. K. Skolem, K. Gödel, Genzen etc.) However sometimes change of erecting the problem itself is more adequate, especially for applied mathematicians (see e. g. BLP).

[1.12] For the engineers and applied mathematicians, the essential object of mathematical theory is regarded to aim the ingenious quantitative treatment of the phenomena, so instead of indulging passively into the problem "what is the continuum essentially ?", we subdivide actively the continuum in finite equal or unequal parts ("cells" or "simplexes" in mathematical words), and then we apply all finite analysis.

[1.13] Then we become able to predict the phenomena under the given boundary conditions (boundary value problems).

[1.14] But it is important to inform that we become able also to determine boundary values inversely from the practical requirements to the quantities of phenomena e. g. electromagnetic field distribution so far as it is consistent with the physical laws, because the number of the unknowns for this problem is already finite in our case, so the solution should be obtained axiomatically by finite process. We would call this backward calculation "realization calculus" or "design calculus" in contrast with the above forward calculus (DKMM. I. : Foreword for Instructors : vii-xv)

[1.15] If necessary, we can investigate the properties relating continuity by taking the limit of the solution by taking the limit of subdivision.

The ordinary analysis can be regarded to be applied repeatedly the idea or process of limit e. g. at the definition of irrational numbers to compute, transiendental functions such as e^x, sin x, sinh x etc., differentiation, integration, expansion in series, numerical approximations etc.. On the contrary to this, the above process of subdivision can be deemed as only one time application of process of limes which converts the continuum into the finite "discretum" by the axiom of Archimedes or cell-subdivision. (WK, CMIA, MAA, SD etc.)

[1.16] Difference calculus, analogue computer and digital computer are usually regarded to become inaccurate in the meaning of δ, ε of limitting operation. We treat here finite laws, but our equations are precise in the above limitting meaning just as the addition theorem of sin x, e^x etc. But of course they have finite accuracy, when they are represented by finite numericals.

[1.17] The functions which we encounter most frequently in daily computation are e^x, sin x, cos x, sinh x etc. These elementary functions including even elliptic functions have all "algebraic" addition theorems, namely for these functions f (x) there exist always certain "algebraic" functions F (x) which satisfy

$$F(f(x+y),\ f(x),\ f(y)) = 0. \tag{1.13}$$

This shows that the value of f (x+y) is obtained from f (x) and f (y) by only purely algebraic computations and it does not need any limitting operations, though these functions f (x) are transiendental and so defined as limits of algebraic functions.

This fact seems to support us to construct algebraic physical laws.

On the contrary to this e. g. Bessel function :

$$f(x) = J_n(x) = 1/\pi \int_0^\pi \cos(x \sin t - nt)\,dt = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!(n+m)!}(x/2)^{n+2m} \tag{1.4}$$

$$J_n(x+y) = \sum_{S=-\infty}^{S=\infty} J_s(x) J_{n-s}(y). \tag{1.5}$$

So it has no algebraic addition formula.

But when we admit to enter bare x and y into the above addition formula :

$$F(f(x+y),\ f(x),\ x,\ y,) = 0.$$

then how many functions are included into these functions which have this addition formula ?

The physical laws which we obtained below are not addition formula themselves, so the extension should be performed to other direction.

[1.18] General trends of analysis in mathematics. As we mentioned above, physical laws are almost represented in the forms of differential equations hitherto.

Influenced by this, also differential equations had been often the principal object of research in analysis in "pure" mathematics. But because the operation "limes" is not at all almighty, algebraic ideas had been increasingly introduced as main steps of solving methods.

1) Utilization of algebraic properties of $p = d/dt$ in "Heaviside's calculus" and symbolic method in general linear differential equations.

2) Indication of simple algebraic properties of integral laws such as Coulomb, Biot-Savart, potential in the vector field analysis by adopting ∇^{-1}, ∇^{-2} as shown in Ohq. Oae (1) (2).

3) Application of algebraic concepts into the analysis e. g. matrices, groups (as Lie group, topological group), ring. (Lie ring, differential ring), ideal (differential ideal), Grassmannian algebra in Cartan's and others' researches for analysis (CLG, CSDE, LAAL, TDS, WCG, RDEAS, RDA etc.)

4) Application of the algebraic topology in the analysis "in the Large", especially in the theory of harmonic fields (HHI, Khr, dKHI etc.)

These are only a global survey and therefore of course quite incomplete and non-precise. Research by difference equations for the differential equations as performed by H. Poincaré in PO. I. iL — cxxix seems to be the most exhaustive to apply algebra and even number theory into the analysis. We owe our way of research much to him in this point.

c) <u>another way which oriented us to this research</u>

[1 9] Many problems of predict calculus in electrical engineering are reduced to the boundary value problems of Maxwell's equations. For these there are already analogue and digital computers, difference calculus and approximation by integral equations (e. g. Hallén's integral equations, SAAT 128—51, UMadt). But various operations and quantities which appear in each step of computation have little physical meaning in general. So these have little connection with the physical and geometrical intuitions for the field distribution.

About their accuracy see [1.16].

[1.20] We constructed a new theory which makes axiomatically possible both the above predict and realization calculus for the Maxwell's electromagnetic field under the name "electromagnetic lattice theory" in 1944 in Oemt (1)—(4) and later "Poincaré process" in 1951 on Op. This idea of substituting a field with a discrete space lattice can be applied to fields of any kind, as well known in analogue computers.

We believe that predict and realization problems under the category of Maxwell's field equations are axiomatically solved or at least given the way to proceed. But it means only about their space character.

In most electromagnetic problems, we treat about time variable "static" or steady state (alternative current). In the latter case, putting the variation about time as $e^{j\omega t}$, $d/dt = j\omega = \lambda$, it becomes quite simply algebra of λ about time parameter.

[1.21] Circuit phenomena are special case of field phenomena and they are pure algebra about space character. Field problem is reduced to the algebra of circuit by lattice.

This suggests us a construction of algebraic theory about time for the transient and steady state phenomena both in circuit and field. It is expected to be able to extend to general physical phenomena. It corresponds symbolically to "x" in

$$\text{field } f(x^1, x^2, x^3) : \text{transient } f(t) = \text{space} : \text{time} = 3-\text{dimension} : 1-\text{dimension}$$
$$= \text{circuit} : x \tag{1 6}$$

or

$$\text{field} : \text{circuit} = \text{transient} : x = \text{continuum} : \text{discretum}. \tag{1.7}$$

[1.22] At the biginning of relativity, similarity of time axis to the space axis was insisted impressively in the 4-dimensional "world". But in the case of space, Poisson's equation is elliptic, on the contrary to this, D'Alembertian equation is hyperbolic type, and the time axis is not so completely symmetric to the space axes. In recent explanation of relativity, rather this difference became to be indicated signicantly.

So the algebraic time theory expected by us is not necessarily similar to the special case of 1-dimension in the pure space equation.

In 1948-9 we tried to construct 4-dimensional electromagnetic lattice theory in the graduate research of Sigeho Nisida in the Tohoku University, but we felt difficulty to interprete physical meaning along time coordinate. So we returned to bigin from time algebra of transient and steady state electric circuits with lumped constants.

[1.23] Poisson's equation and D'Alembertian equation are both partial differential equations, but on the contrary to these, equations of Newtonian mechanics are generally ordinary differential ones So if one succeed to extend time algebra to the Netwonian mechanics, they may become to a kind of algebraic theory.

A simple survey over the vector analysis in 3-dimension shows also this fact, namely Newtonian mechanics appears just after the vector algebra under the short preparation of 1-parameter differentiation and integration of vectors. It does not need gradient, divergence and rotation, namely algebra of ∇, ∇^2, ∇^{-1}, ∇^{-2}, $\Box$, $\Box^2$ etc (Ohq)

d) a survey of general mathematical physics.

[1.24] Now we would turn to a survey to general mathematical physics, because this branch of science is often deemed to be the most progressive and brave in adopting new mathematical ideas and device, and it has been really often eminent models of applications of mathematics. As mentioned above in [1.18], quantitative laws have been represented almost in the forms of differential equations, namely as local and instantaneous properties "in the small". The main objects of physical mathematics were differential equations e. g. in Whittaker-Watson, Courant-Hilbert, Frank-Mises etc. But for the requests for the properties "in the large" or at least "in finite", (abstract) algebraic concepts are gradually introduced in mathematical physics, for example,

matrix calculus by Heisenberg, Pauli and Dirac,

application of groups on the quantum theory by H. Weyl,

introduction of group theoretic standpoint to Lorentz transformation in the relativity,

application of lattice theory to the quantum theory by G. Birkhoff and J. von Neumann (BNlqm).

These seem fragmental applications of algebraic concepts. A more systematic description seems to be possible, but not by the present standardized mathematics.

Recently developed synthetic tendency in mathematics (e. g. algebraic topology, algebraic geometry based on the rigid abstract algebra, two or three valued calculus of the lattice theory (BLT) etc.) shows orientation.

[1.25] If we stand on cosmology that all existences reduce to elementary particles, it will be enough to know and manage the behaviours of these discrete particles by laws between particles, and it may not be necessary to investigate "continuous" field distribution outsides of the particles, this leads us rather to algebra than analysis of continuum.

[1.26] Theory of crystal lattice is another example.
Carnot's cycle is an algebraification of heat engine, because if we adopt temperature and heat as abscissa and ordinate, it becomes representation of closed curve a set of rectangles in order to separate adiabatic and isothermal changes.

Relation of potential and charge of conductors in electrostatic field is a typical introduction of algebra into "the field".

Measurement of magnetic flux by ballistic galvanometer owes to integral relation of flux and ballistic throw about time variable.

We have already some integral concepts in time such as momentum, impulse and even time-flux (WAD. 17).

e) models preferable to refer.

[1.27] Reduction of electromagnetic field to the algebra of lattice structure of interlinked electric and megnetic networks (Oae (1), (2), Op and [1.20]) by the theorem of Gauss and Stokes in the modern theory of harmonic integral

(HHI, Khr, dKHI etc.) by previously descending the order of differential equations to simultanceous equations of the first order. Partial diff. equa. of higher order → sim. part. diff. equ. of the first order → subdivision of the domain under consideration → app. of Gauss and Stokes → relation of integral quantities between cells (simplexes) → finite algebraic laws.

Or more shortly,

diff. equ. → (above process) → difference equations → change of determination of medium constants → algebraic laws.

[1 28] It is a wellknown fact that linear difference equations with constant coefficients are satisfied by the same exponential functions to the corresponding differerential equations. So if we regard the solution of differential equations as values of experiments, and we determine coefficients of difference equations from "experiments", we obtain precise values by these difference equations which do not contain any approximations caused by their finiteness, or by not taking the limit.

The following laws are fundamentally based upon this logic.

Chapter Ⅱ. Preliminaries

a) Mathematical preliminaries

[2.1] Time. We adopt at first Newtonian "absolute" time, namely we regard time as uniform flow. Afterwards it becomes clear that multi-clock idea in the meaning of relativity by Einstein, Dirac, Tomonaga etc. can be easily treated without differential and integral calculus. Thus physical substance of relativity seems to become clear by avoiding troublesome "differential geometry" or other mathematical complicacy. Thus time flow is shown at first defined as "simply ordered set" (BLT 10 Def. 6) or "chain". It is a 1−dimensional complex C^1 homeomorphic with a single straight line, but it may be extended to any complicated C^1 for multi-clock theory.

[2.2] We subdivide the time-chain C^1 at first in uniform interval T sec. e. g. in 1 millisecond, $T=10^{-3}$, 1 microsecond, $T=10^{-6}$, 1 hour, $T=60^2$, 1 year, $T=60^2\times 24\times 365$ etc If necessary, we can subdivide the time in non-uniform interval, in certain case simultaneously in $T=1$ ms for the first network, or branch, $T=1\mu$s for the second network or branch etc. Thus we obtain algebraic method of treating multi-clock system. But we do not treat such multi-clock system in this paper.

[2.3] Nomenclature

At first we call the instant in which we bigin the observation "0−instant" (timely 0−cell or timely 0−simplex in timely 1−complex), we divide time in equal interval T (period, timely 1−cell, timely 1−simplex), and call the first period just after the 0—instant also 0—period as in Fig. .1

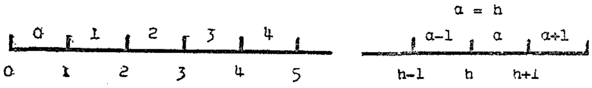

Fig. 1

[2.4] If we write a definite not only Riemannian, but also Stieljes' or Lebesgue's) integral of any function f(t) in period α $I_\alpha f(t)$, namely

$$I_\alpha f(t) \overset{D}{=} \int_{Th}^{hT+T} f(t)dt, \quad \text{where } \overset{D}{=} \text{ shows difinition formula.} \tag{2.1}$$

then for mean value $f_{\alpha m}$,

$$f_{\alpha m} = \frac{1}{T}\int_{hT}^{hT+T} f(t)\,dt \tag{2.2}$$

$$= \frac{1}{T} I_\alpha f(t) \tag{2.3}$$

So if we note initial (instant) value, central point value and final value in a period respectivey $f_{\alpha i}$: $f_{\alpha c}$ and $f_{\alpha f}$, then for the above divison in Fig. 1.

$$f_{\alpha m} = f(hT+\theta T), \quad 0 \leqq \theta \leqq 1; \tag{2.4m}$$

$$f_{\alpha i} = f(hT); \tag{2.4i}$$

$$f_{\alpha c} = f(hT+T/2); \tag{2.4c}$$

$$f_{\alpha f} = f(hT+T); \tag{2.4f}$$

and

$$I_\alpha f(t) = T\, f_{\alpha m}, \tag{2.5m}$$

$$= T\,\lambda_{\alpha i}\, f_{\alpha i}, \tag{2.5 i}$$

$$\lim_{T\to 0} \lambda_{\alpha i} = 1, \tag{2.5λi}$$

$$= T\,\lambda_{\alpha c}\, f_{\alpha c}, \tag{2.5 c}$$

$$\lim_{T\to 0} \lambda_{\alpha c} = 1, \tag{2.5λc}$$

$$= T\,\lambda_{\alpha f}\, f_{\alpha f}, \tag{2.5 f}$$

$$\lim_{T\to 0} \lambda_{\alpha f} = 1, \tag{2.5λf}$$

$\lambda_{\alpha i}$, $\lambda_{\alpha c}$ and $\lambda_{\alpha f}$ are generally functions about α, but for costant a,

$$I_\alpha a = \int_{hT}^{hT+T} a\,dt = aT, \tag{2.6}$$

$$f_{\alpha m} = f_{\alpha i} = f_{\alpha c} = f_{\alpha f} = a, \quad \lambda_{\alpha i} = \lambda_{\alpha c} = \lambda_{\alpha f} = 1, \tag{2.7}$$

for (complex) exponential function :

$$I_\alpha\, Ae^{at} = \int_{hT}^{hT+T} Ae^{at}\,dt = Ae^{hT}(e^{T}-1)/a \tag{2.8}$$

$$f_{\alpha i} = Ae^{hT}, \quad \lambda_{\alpha i} = (e^{T}-1)/aT, \tag{2 9i}$$

$$f_{\alpha c} = Ae^{h+T/2}, \quad \lambda_{\alpha c} = (e^{T/2}-e^{-T/2})/aT, \tag{2.9c}$$

$$f_{\alpha f} = Ae^{hT+T}, \quad \lambda_{\alpha f} = (1-e^{-T})/aT. \tag{2.9f}$$

So in all cases, they are indepent to h, in equal division T=const, so they become constants. In linear networks of L, R, C, their phenomena are linear combinations of exponential functions including complex variable. So these cases are all included in the above condition. Therefore, so far as we treat linear constant networks, we can treat them within the above condition.

[2.5] Way to proceed.

Since a. c. theory can be regarded to be obtained by elimination of magnetic quantities from electromagnetic networks in case of absence of mmf. and magnetic poles, if we want time algebra directly corresponding to the a. c. theory, it seems desirable to build a first time algebra of electromagnetic networks, and then to eliminate magnetic quantities analogous to the above non-time-algebra case. But in this paper we select a short-cut to proceed directly to enter into costruction of time-algebra of a. c. theory, because a. c. theory seems far more popular than the idea of generally complicatedly interlinked electromagnetic networks.

b) physical preliminaries

[2.6] time algebraic concepts for active energy sources, and also passive drops. Corresponding to impulse $\int f dt$ in collision of Newtonian mechanics, we use time integral of active electromotive forcce e(t) and passive voltage drops u(t), and also magnetomotive force c(t) and magnetic ampere turns drop h(t):

$I_\alpha e(t) = e_\alpha$ weber : (active) electric impulse, energy source,

$I_\alpha u(t) = u_\alpha$ weber : (passive) electric shock, energy consumption,

$I_\alpha c(t) = c_\alpha$ coulomb : (active)magnetic impulse, energy source,

$I_\alpha h(t) = h_\alpha$ coulomb : (passive) magnetic shock, energy consumption.

Similarly, for active electric current source s(t) ampere, passive current through the element i(t),

$I_\alpha s(t) = s^\alpha$ coulomb : (active) electric input charge,

$I_\alpha i(t) = i^\alpha$ coulomb : (passive) electric charge passed in α-period.

As $$i = \dot{q}, \tag{+2.10}$$

i^α can be written also q^α namely

$$I_\alpha i = i^\alpha = I_\alpha \dot{q} = q^\alpha. \tag{+2.11}$$

In order to enable summation convention, we use also contravariant indices. If we define total charge passed from the O-instant to h-instant as q^h, then q^α can be expressed as a difference of them, e. g. in Fig. 1.

$$q^\alpha = q^{h+1} - q^h, \tag{+2.12}$$

but in general in matrix :

$$\boxed{q^\alpha = d^\alpha_h q^h.} \tag{+2.13}$$

The matrix d^α_h corresponds to time-differential $\partial/\partial t$. This holds also for charge in a condenser.

Dually, for voltage drop u(t) of Faraday's electromagnetic induction in an inductance, where ϕ represents its linked magnetic flux, then

$$u = \dot{\phi}. \tag{−2.10}$$

So,

$$I_\beta u = u_\beta = I_\beta \dot{\phi} \overset{D}{=} \phi_\beta, \tag{−2.11}$$

$$\phi_\beta = \phi_i - \phi_{i-1} \quad \text{or} \quad \phi_{i+1} - \phi_i \tag{−2.12}$$

in general,

$$\boxed{\phi_\beta = d^i_\beta \, \phi_i} \qquad (-2.13)$$

Matrix d^i_β is not always equal to d^α_h, though in certain cases, it can be made identical.

[2.7] time-algebraic laws for resistance.

If we apply I_α for

$$u = r\dot{q}, \qquad (+2.14) \qquad \dot{q} = gu, \qquad (-2.14)$$

assuming r constant during this period, then

$$u_\beta = I_\beta u = I_\beta r\dot{q} = r\, I_\beta \dot{q} = rq^\beta ,$$

or more generally

$$u_\beta = r_{\beta\alpha} q^\alpha \ , \qquad (+2.15)$$

similarly

$$q^\alpha = g^{\alpha\beta} u_\beta \ , \qquad (-2.15)$$

If these two relations represent an identical fact,

$$g^{\alpha\gamma}\, r_{\gamma\beta} = r_{\beta\gamma}\, g^{\gamma\alpha} = \delta^\alpha_\beta \ : \text{Kronecker's symbol.} \qquad (\pm 2.16)$$

$r_{\beta\alpha}$ $(\beta \neq \alpha)$ represent that an electric impulse of u_β weber in a period β causes charge displacement q^α coulomb in a period α, where ordinarly $\beta > \alpha$, If $\alpha < \beta$ pass charge q^α causes u_β shock to β—period.

If there exist

$$g^{\kappa\lambda} \neq 0 \ (\kappa \neq \lambda) \text{ as in a triode valve},$$

$$i^\kappa = g^{\kappa\lambda}\, u_\lambda , \qquad u_\lambda = r_{\lambda\kappa}\, i^\kappa , \qquad \kappa, \lambda : \text{number of branches,}$$

Also we can obtain the same (-2.15) by giving the single numbers α and β all over the periods of all branches, $\alpha \times \kappa$.

In case of resistance in parallel to an inductance, similarly from

$$i = g\,\dot{\phi}, \qquad (+2.17), \qquad \dot{\phi} = ri \qquad (-2.17)$$

we obtain

$$\boxed{i^\alpha = \dot{q}^\alpha = g^{\alpha\beta}\, \phi_\beta} \quad (+2.18) \qquad \boxed{\phi_\alpha = r_{\beta\alpha}\, q^\alpha = r_{\beta\alpha} i^\alpha} \quad (-2.18)$$

(± 2.15) and (± 2.18) are mutually dual.

[2.8] laws of capacity.

By applying I_β for

$$u = sq \ , \qquad (+2.19) \qquad q = cu \ , \qquad (-2.19)$$

where s and c are constant in β—period, then we obtain

$$u_\beta = I_\beta\, u = I_\beta\, sq = sI_\beta\, q \ ; \qquad (2.20)$$

from (2.5i) in [2.4],

$$u_\beta = sT\, \lambda_{\beta i}\, q^{\beta i} \qquad (2.21)$$

For Fig. 1,

$$u_\beta = sT\, \lambda_{\beta i}\, q^h \qquad (2.22)$$

Here we would consider physical meaning of right hand side. Resultant resistance $\dot{r}$ of a material of lenfth s, cross section f and resistivity ρ is

$$r = \rho s/f. \tag{2.23}$$

If the current distribution is not uniform, then by adopting a correcting factor λ $(\lim_{s,f\to 0} \lambda = 1)$

$$r = \lambda \rho s/f. \tag{2.24}$$

In this case r is determined not from this but from measurement of u and i in $u = ri$, inversely λ is determined from u, i, ρ, s and f.

ρ is a resultant resistance $\rho = r$ where $f = s = 1$ and uniform current distribution. So r is a product of ρ, equalizing factor λ and space-dimensional factor s and 1/f.

Now if we put

$$sT\lambda_{\beta i} = s_{\beta h}, \tag{+2.25}$$

this corresponds just to

$$\lambda \to \lambda_{\beta i}, \quad s/f \to T, \quad \rho \to s, \quad r_{\lambda\kappa} \to s_{\beta h}$$

and it becomes clear that $s_{\beta h}$ is a new physical medium constant including time progress and so depending upon T. Usual elastance s is $s_{\beta h}$ of one second just as ρ is r of unit volume.

After all we get

$$\boxed{u_\beta = s_{\beta h} q^h} \quad (+2.26) \qquad \boxed{q^h = c^{h\beta} u_\beta} \quad (-2.26)$$

$$s_{\beta j} c^{j\alpha} = \delta_\beta^\alpha, \qquad c^{h\gamma} s_{\gamma i} = \delta_i^h \tag{±2,27}$$

$s_{\beta h}$ and $c^{h\beta}$are determined experimentally from measurements of u_β and q^h just as in r in (2.24). So also it can be extended to variable capacity as equivalent $s_{\beta h}$ and $c^{h\beta}$, where these are determined from measurents of the both quantities u_β and q^h.

[2.9] laws of inductance.

Dually to [2.8] from

$$i = k\phi \quad (+2.28) \qquad \phi = li \quad (-2.28)$$

We obtain

$$\boxed{i^\alpha = k^{\alpha i} \phi_i} \quad (+2.29) \qquad \boxed{\phi_i = l_{i\alpha} i^\alpha} \quad (-2.29)$$

$$k^{\alpha i} = kT\lambda_{\alpha i}, \tag{-2.25}$$

$$k^{\alpha j} l_{j\beta} = \delta_\beta^\alpha, \qquad l_{i\gamma} k^{\gamma h} = \delta_i^h. \tag{±2.30}$$

[2.30] Conclusion.

For capacity, we get from (+2.13) and (−2.26)

$$q^\alpha = d_h^\alpha q^h = d_h^\alpha c^{h\beta} u_\beta, \tag{+2,31}$$

dually for inductance, we get from (−2.13) and (2.29)

$$\phi_\beta = d_\beta^i \phi_i = d_\beta^i l_{i\alpha} i^\alpha, \tag{-2.31}$$

For element of series of L, R and C, applying I_β to

$$u = \dot{\phi} + r\dot{q} + sq, \tag{+2.32}$$

$$k\phi - \dot{q} = 0 \quad \text{or} \quad \phi - l\dot{q} = 0, \tag{+2.33}$$

we get

$$u_\beta = \phi_\beta + r_{\beta\alpha}\, q^\alpha + s_{\beta h}\, q^h , \tag{+2.34}$$

$$k^{\alpha i}\, \phi_i - q^\alpha = 0 \;, \quad \text{or} \quad \phi_i - l_{i\alpha}\, q^\alpha = 0 . \tag{+2.35}$$

$$\therefore \quad u_\beta = d^i_\beta\, l_{i\alpha}\, d^\alpha_h\, q^h + r_{\beta\alpha}\, d^\alpha_h\, q^h + s_{\beta h}\, q^h \tag{+2.36}$$

This corresponds just to

$$u_\lambda = \frac{d}{dt}\left(l_{\lambda\kappa} \frac{dq^\kappa}{dt} \right) + r_{\kappa\lambda} \frac{dq^\kappa}{dt} + s_{\lambda\kappa}\, q^\kappa . \tag{+2.37}$$

For element of parallel of C, G, K, from

$$i = \dot{q} + g\,\dot{\phi} + k\phi , \tag{−2.32}$$

$$s\,q - \dot{\phi} = 0 \qquad \text{or} \qquad q - c\,\dot{\phi} = 0 \tag{−2.33}$$

we get

$$i^\alpha = q^\alpha + g^{\alpha\beta}\, \phi_\beta + k^{\alpha i}\, \phi_i , \tag{−2.34}$$

$$s_{\beta h}\, q^h - \phi_\beta = 0 \,, \quad \text{or} \quad q^h - c^{h\beta}\, \phi_\beta = 0 \tag{−2.35}$$

$$i^\alpha = d^\alpha_h\, c^{h\beta}\, d^i_\beta\, \phi_i + g^{\alpha\beta}\, d^i_\beta\, \phi_i + k^{\alpha i}\, \phi_i . \tag{−2.36}$$

This corresponds just to

$$i^\kappa = \frac{d}{dt}\left(c^{\kappa\lambda} \frac{d\phi_\lambda}{dt} \right) + g^{\kappa\lambda} \frac{d\phi_\lambda}{dt} + k^{\kappa\lambda}\, \phi_\lambda . \tag{−2.37}$$

If we give continuity or constraint for q^h or ϕ_i, just as 1−Kirchhoff's law (Oae (1)) :

$$D^\alpha_\kappa\, i^\kappa = D^\alpha_\kappa\, s^\kappa = s^\alpha$$

in ordinary networks, equivalent of 2−Kirchhoff's law is to be obtained and as a complete system, whole theory may correspond again to flat-subspace theory. But we leave these all in chance of future. One reason is that it should be performed more simply by eliminating magnetic qnantity from time-algebra of electromagnetic networks as mentioned above in[2.5]. But here we would describe fundamental equation of electromagnetic networks as starting base in the following [2.11]. We would mean these also as a supplement to Oae (2). About notations and the details of equations in [2.11], refer Oae (2).

[2.11] fundamental laws of interlinked electromagnetic networks.

O−Kh. :

$$\{ d^{\alpha_1}_{h_1}\, t^{h_1} = 0, \quad d^{\alpha_2}_{h_2}\, m^{h_2} = 0 . \tag{±2.38′}$$

1−Kh. :

$$\begin{cases} D^{\alpha_1}_{\kappa_1}\, k^{\kappa_1} = D^{\alpha_1}_{\kappa_1}\, t^{\kappa_1} = t^{\alpha_1} , & (+2.38) \\ D^{\alpha_2}_{\kappa_2}\, b^{\kappa_2} = D^{\alpha_2}_{\kappa_2}\, m^{\kappa_2} = m^{\alpha_2} . & (+2.39) \end{cases}$$

2−Kh. :

$$\begin{cases} R^{\lambda_1}_{q_1}\, u_{\lambda_1} = R^{\lambda_1}_{q_1}\, e_{\lambda_1} = e_{q_1} , & (-2.38) \\ R^{\lambda_2}_{q_2}\, h_{\lambda_2} = R^{\lambda_2}_{q_2}\, c_{\lambda_2} = c_{q_2} . & (-2.39) \end{cases}$$

$$\begin{cases} k^{\kappa_1} - t^{\kappa_1} = K^{\kappa_1} = -\, T''^{\kappa_1} = R^{\kappa_1}_{p_1}\, J^{p_1} = -\, R^{\kappa_1}_{p_1}\, T^{p_1} , & (+2.40) \\ b^{\kappa_2} - m^{\kappa_2} = B^{\kappa_2} = -M''^{\kappa_2} = R^{\kappa_2}_{p_2}\, J^{p_2} = -\, R^{\kappa_2}_{p_2}\, M^{p_2} . & (+2.40) \end{cases}$$

$$\begin{cases} u_{\lambda_1} - e_{\lambda_1} = U_{\lambda_1} = -\, E'_{\lambda_1} = D^{b_1}_{\lambda_1}\, V_{b_1} = -D^{b_1}_{\lambda_1}\, E_{b_1} & (-3\,40) \\ h_{\lambda_2} - c_{\lambda_2} = H_{\lambda_2} = -\, C'_{\lambda_2} = D^{b_2}_{\lambda_2}\, V_{b_2} = -D^{b_2}_{\lambda_2}\, C_{b_2} & (-2.41) \end{cases}$$

$$\dot{k}^{\kappa_1} = d^{\kappa_1} + q^{\kappa_1} \tag{2.42}$$

$$d^{\kappa_1} = c^{\kappa_1 \lambda_1} u_{\lambda_1}, \quad u_{\lambda_1} = s_{\lambda_1 \kappa_1} d^{\kappa_1}, \qquad (+2.43)\ (-2.43)$$

$$\dot{q}^{\kappa_1} = g^{\kappa_1 \lambda_1} u_{\lambda_1}, \quad u^{\lambda_1} = r_{\lambda_1 \kappa_1} \dot{q}^{\kappa_1} \qquad (+2.44)\ (-2.44)$$

$$\dot{k}^{\kappa_1} = \dot{d}^{\kappa_1} + \dot{q}^{\kappa_1} = p\, c^{\kappa_1 \lambda_1} u_{\lambda_1} + g^{\kappa_1 \lambda_1} u_{\lambda_1} = y^{\kappa_1 \lambda_1} u_{\lambda_1}, \quad p = d/dt. \qquad (+2.45)$$

($\pm 2.43 \sim 5$) hold only for solely electric networks. If we take account of mutual action of both networks in u_{λ_1} and h_{λ_1}, then using the above $z_{\lambda_1 \kappa_1}$, we have

$$u_{\lambda_1} = z_{\lambda_1 \kappa_1} p\, k^{\kappa_1} + p\, n_{\lambda_1 \kappa_2} b^{\kappa_2}, \qquad (-2.46)$$

$$h_{\lambda_2} = -p\, n_{\lambda_2 \kappa_1} k^{\kappa_1} + w_{\lambda_2 \kappa_2} b^{\kappa_2}. \qquad (+2.46)$$

Inversely if we represent k and b in terms of u and h, we obtain

$$\left.\begin{aligned} k^{\kappa_1} &= a^{\kappa_1 \lambda_1} u_{\lambda_1} + a^{\kappa_1 \lambda_2} h_{\lambda_2}, \\ b^{\kappa_2} &= a^{\kappa_2 \lambda_1} u_{\lambda_1} + a^{\kappa_2 \lambda_2} h_{\lambda_2}, \end{aligned}\right. \qquad (-2.46)$$

where

$$\begin{vmatrix} z_{\lambda_1 \kappa_1} p & p n_{\lambda_1 \kappa_2} \\ -p n_{\lambda_2 \kappa_1} & w_{\lambda_2 \kappa_2} \end{vmatrix} = \begin{vmatrix} a_{\lambda_1 \kappa_1} & a_{\lambda_1 \kappa_2} \\ a_{\lambda_2 \kappa_1} & a_{\lambda_2 \kappa_2} \end{vmatrix}, \qquad (2.47)$$

$$\begin{vmatrix} a^{\kappa_1 \mu_1} & a^{\kappa_1 \mu_2} \\ a^{\kappa_2 \mu_1} & a^{\kappa_2 \mu_2} \end{vmatrix} \begin{vmatrix} a_{\mu_1 \lambda_1} & a_{\mu_1 \lambda_2} \\ a_{\mu_2 \lambda_1} & a_{\mu_2 \lambda_2} \end{vmatrix} = \begin{vmatrix} \delta^{\lambda_1}_{\kappa_1} & 0 \\ 0 & \delta^{\kappa_2}_{\lambda_2} \end{vmatrix} \qquad (2.48)$$

Besides these,

$$\phi_{q_1} = R^{\lambda_1}_{q_1} A_{\lambda_1}, \quad (+2.49) \qquad A_{\lambda_1} = n_{\lambda_1 \kappa_2} b^{\kappa_2}, \quad (+2.50)$$

$$\Phi_{q_2} = R^{\lambda_2}_{q_2} a_{\lambda_2} \quad (-2.49) \qquad a_{\lambda_2} = n_{\lambda_2 \kappa_1} k^{\kappa_1} \quad (-2.50)$$

are usually utilized. For example equations for loop unknown quantities,

$$R^{\lambda_1}_{q_1}(z_{\lambda_1 \kappa_1} p\, R^{\kappa_1}_{p_1} J^{p_1} + p\, n_{\lambda_1 \kappa_2} R^{\kappa_2}_{p_2} J^{p_2}) = R^{\lambda_1}_{q_1}(e_{\lambda_1} - z_{\lambda_1 \kappa_1} t^{\kappa_1} - p\, n_{\lambda_1 \kappa_2} m^{\kappa_2}),$$

$$R^{\lambda_2}_{q_2}(-p\, n_{\lambda_2 \kappa_1} R^{\kappa_1}_{p_1} J^{p_1} + w_{\lambda_2 \kappa_2} R^{\kappa_2}_{p_2} J^{p_2}) = R^{\lambda_2}_{q_2}(c_{\lambda_2} + p\, n_{\lambda_2 \kappa_1} t^{\kappa_1} - w_{\lambda_2 \kappa_2} m^{\kappa_2}).$$

If there are

$$m^{\kappa_2} = 0, \quad c_{\lambda_2} = 0, \quad n_{\lambda_2 \kappa_1} t^{\kappa_1} = 0, \text{ even in case of } t^{\kappa_1} \neq 0, \qquad (2.51)$$

then mmf. is caused only from linked flux diminution. In this case

$$A_{\lambda_1} = n_{\lambda_1 \kappa_2} b^{\kappa_2} = n_{\lambda_1 \kappa_2} R^{\kappa_2 \lambda_2} \dot{a}_{\lambda_2} = n_{\lambda_1 \kappa_2} R^{\kappa_2 \lambda_2} n_{\lambda_2 \kappa_1} \dot{k}^{\kappa_1} \qquad (2.52)$$

$$\overset{D}{=} l_{\lambda_1 \kappa_1} \dot{k}^{\kappa_1}. \qquad (2.53)$$

[2.12] Product of impluse e_β and q^α e. g. $e_1 q^1$ is not energy in capacity but action of Hamilton. It is the same also in magnetic case.

Chapter Ⅲ. Examples

[3.30] Introduction

Here we give a few examples in simplest cases.

[3.1] discharge of condenser through a resistance.

$$r\dot{q} + s q = 0 \tag{3.1}$$

By applying I_α to this, we get

$$r_{\beta\alpha} q^{\alpha} + s_{\beta h} q^{h} = r_{\alpha\beta} d^{\alpha}_{h} q^{h} + s_{\beta h} q^{h} = 0\,, \tag{3.2}$$

If we assume $\underset{\beta}{g}$ and $\underset{\beta}{s}$ are constant and equal to $\underset{0}{g}$ and $\underset{0}{s}$, then in case of relation of instant period in Fig. 1.

$$\underset{0}{r}\,(q^{h+1} - q^{h}) + \underset{0}{s}\,q^{h} = 0\,, \tag{3.3}$$

$$\therefore \quad q^{h+1} = (1-\underset{0}{g}\,\underset{0}{s})\,q^{h}\,.$$

$$q^{1} = (1-\underset{0}{g}\,\underset{0}{s})q^{0}\,, \qquad q^{2}=(1-\underset{0}{g}\,\underset{0}{s})q^{1}=(1-\underset{0}{g}\,\underset{0}{s})^{2}q^{0}\,, \qquad q^{3}=(1-\underset{0}{g}\,\underset{0}{s})^{3}q^{0}, \cdots$$

$$q^{h} = (1-\underset{0}{g}\,\underset{0}{s})^{h}q^{0}\,, \qquad q^{0} : \text{initial charge}\,, \tag{3.4}$$

If we use the following relations which connect with usual expression.

$$\underset{0}{g} = g\,, \qquad \underset{0}{s} = sT\lambda\,, \qquad h = t/T\,, \tag{3.5}$$

then

$$q^{h} = q(t) = q(hT) = q^{0}(1-gsT\lambda)^{t/T} \tag{3.6}$$

This coincides with

$$q(t) = q^{0}\,e^{-gst}\,, \tag{3.7}$$

if

$$(-gsT\lambda)^{1/T} = e^{-gs}\,. \tag{3.8}$$

This holds good, because coefficient gs of (3. 4) is determined by experimental values. λ in (3.8) is indirectly determined from (3.8) through "experiment".

In conclusion, (3.3) is the requested fundamental algebraic law for this case without approximation. (3.4) is obtained by successive substitution, so purely algebraically.

Notice. If we take limit of (3·6) $T \to O$,

$$\lim_{T\to 0}(1-gsT\lambda)^{1/T} = \lim_{\mu\to\infty}\left(1-\frac{gs\lambda}{\mu}\right)^{\mu} = \lim_{\mu\to\infty} e^{-gs\lambda} = e^{-gs} \tag{3.9}$$

because $\lim\limits_{T\to 0}\lambda = 1$, so this coincides with (3·7). This is a new approach of limes calculus attended by physical image.

[3.2] Charge of C by emf. e(t) through a resistance.

$$r\dot{q} + s q = e(t)\,, \tag{3.10}$$

e(t) may be any arbitrary function of t. By operating I_β,

$$\underset{0}{r}(q^{h+1}-q^{h})+\underset{0}{s}q^{h}=e_\beta \tag{3.11}$$

if $e_0 = e_1 = e_2 = \cdots = e_\beta$, then

$$q^{h} = \lambda^{-1}ce_0/T+(q^{0}-\lambda^{-1}ce_0/T)(1-\underset{0}{g}\,\underset{0}{s})^{h}\,. \tag{3.12}$$

fi

$$q^{0} = 0\,, \quad q^{h}=(\lambda^{-1}\,ce_0/T)\,\{1-(1-\underset{0}{g}\,\underset{0}{s})^{h}\}\,, \tag{3.13}$$

Even in case e_β are all different, we can also obtain q^h by purely algebraic process.

[3.3] Decade of magnetic flux by a resistance. This is dual of (3.1). So quite similarly

$$\dot{\phi} + ri = \dot{\phi} + rk\phi = 0\,, \tag{3.14}$$

$$\phi_{i+1} - \phi_i + \underset{0}{r}\underset{0}{k}\,\phi_i = 0\,, \tag{3.15}$$

$$\phi_{i+1} - (1 - \underset{0}{r}\underset{0}{k})\,\phi_i = 0\,. \tag{3.16}$$

$$\phi_1 = (1 - \underset{0}{r}\underset{0}{k})\,\phi_0\,, \tag{3.17}$$

$$\phi_2 = (1 - \underset{0}{r}\underset{0}{k})\,\phi_1\,, \tag{3.18}$$

$$\phi_i = (1 - \underset{0}{r}\underset{0}{k})^i\,\phi_0\,, \tag{3.19}$$

$$= (1 - r\,kT\lambda)^i\phi_0\,, \tag{3.20}$$

[3.4] Magnetization through resistance.

$$\dot{\phi} + r\,i = \dot{\phi} + r\,k\,\phi = e(t)\,, \tag{3.21}$$

$$\phi_{i+1} - \phi_i + \underset{0}{r}\,\underset{0}{k}\,\phi_i = e_\beta \tag{3.22}$$

Further, see the dual [3.2].

[3.5] Oscillation of L and C.

$$s\,q + \dot{\phi} = 0\,, \qquad k\,\phi - \dot{q} = 0 \tag{3.23) (3.24}$$

At operating I_β to these, in order to bring the result a little more simply, we adopt $\lambda_{\beta i}$ for q^h, $\lambda_{\beta f}$ for ϕ_h, then

$$I_\beta\ (3{,}23):\ \ \underset{0}{s}\,q^h + \phi_{h+1} - \phi_h = 0\,, \tag{3.25}$$

$$I_\beta\ (3{,}24):\ \ \underset{0}{k}\,\phi_h - q^h + q^{h-1} = 0\,, \tag{3.26}$$

Putting (3.26) into (3.25), we get

$$q^{h+1} - 2(1 - \underset{0}{k}\,\underset{0}{s}/2)\,q^h + q^{h-1} = 0\,. \tag{3.27}$$

If q^0 and q^1 are given from experiment, then q^h is algebraically expressed by them.

That (3.27) satisfies experiment, namely satisfies the solution of differential equation is easily verified by substiutting

$$q^h = A \sin \Omega\, h,$$

namely, then left side of (3.27) becomes to

$$A \sin \Omega\, h\ (\cos \Omega - 1 + \underset{0}{k}\underset{0}{s}/2),$$

So Ω is determined from

$$\cos \Omega = 1 - \underset{0}{k}\underset{0}{s}/2\,. \tag{3.29}$$

[3.6] L, R and C in series.

$$u:\quad s\,q + r\,\dot{q} + \phi = 0\,, \tag{3.30}$$

$$i:\quad k\,\phi - \dot{q} = 0\,, \tag{3.31}$$

$$I_\beta\,(3.30):\quad \underset{0}{s}\,q^h + \underset{0}{r}(q^{h+1} - q^h) + \phi_h - \phi_{h-1} = 0\,, \tag{3.32}$$

$$I_\beta\,(3.31):\quad \underset{0}{k}\,\phi_h - q^{h+1} + q^h = 0 \tag{3.33}$$

$$\therefore\quad (1 + \underset{0}{k}\,\underset{0}{r})\,q^{h+1} - 2(1 + \underset{0}{k}\,\underset{0}{r}/2 - \underset{0}{k}\,\underset{0}{s}/2)\,q^h + q^{h-1} = 0\,, \tag{3.34}$$

[3.7] C, G and K in parallel.

$$\text{i}: \quad k\phi + g\dot{\phi} + \dot{q} = 0, \tag{3.35}$$

$$\text{u}: \quad sq - \dot{\phi} = 0 \tag{3.36}$$

This is dual of [3.6].

$$\underset{0}{k}\,\phi_i + \underset{0}{g}(\phi_{i+1} - \phi_i) + q^i - q^{i-1} = 0, \tag{3.37}$$

$$\underset{0}{s}\,q^i - \phi_{i+1} - \phi_i = 0,$$

$$\therefore \quad (1 + \underset{0}{g}\,\underset{0}{s})\,\phi_{i+1} - 2(1 + \underset{0}{g}\,\underset{0}{s}/2 - \underset{0}{k}\,\underset{0}{s}/2)\,\phi_i + \phi_{i-1} = 0, \tag{3.38}$$

Chapter IV Conclusion.

[4] We have continued to build a time-algebraic laws in simplest cases. Its name may be algebraic history of electric networks, electric calendar, electric time circuit etc. Concept of 4—dimensional hyperbolic space of relativity seem to be usefull for general case.

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This is alphabetically ordered in abbreviations.

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電気回路の時変数の代数的基本法則

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概要　L, C, R の電気回路に電圧或は電流で与えたインパルスが不規則に入つた場合の現象の瞬間値を純代数的に求める理論の構成の可能性の証明，方針，原理，必要な新しい物理概念，一般法則，簡単な例を述べ，可能性を簡単な場合に実証した。

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